

ALGEBRA II



SUBJECTS BOOK

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29¢

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GRADE 12

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PART II

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Room 310

Algebra II

PART II

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Simple number relation problems

KEEP IN MIND

If "n" is a certain number the following statements are translated into Algebra in the following manner:

- 1) 6 more than 3 times a certain number... $3n + 6$
- 2) 7 less than 3 times a certain number... $3n - 7$
- 3) Three times a certain number decreased by 4... $3n - 4$
- 4) Five subtracted from 3 times a certain num... $3n - 5$
- 5) In solving a certain problem we let the unknown "n" equal to whatever follows the word "than" or the second word "as" in the problem.

PROBLEMS

- 1) If 2 is added to 3 times a number the result is 10 decreased by 9 times a number. Find the number. Let $n =$ the number

$$3n + 2 = 10 - 9n$$

$$3n + 9n = 10 - 2$$

$$12n = 8$$

$$n = \frac{2}{3} \text{ the number}$$

- 2) The first of two numbers is 2 more than 3 times the second. If the first number is increased by one the result is equal

to four times the second number. Find the numbers.

$$\left. \begin{aligned} \text{Let } s &= \text{the second} \\ 3s + 2 &= \text{the first} \end{aligned} \right\} \text{outline}$$

$$3s + 2 + 1 = 4s$$

$$3s - 4s = -1 - 2$$

$$+s = +3 \text{ The second}$$

$$3s + 2 = 11 \text{ The first}$$

Problems on separating a number into two parts

KEEP IN MIND

- I. To separate a number into two parts we
 Let s = the smaller part (or one part)
 The number minus s = the larger part (or other part)
- II. If we know the sum of two numbers we
 Let s = the smaller number (or one number)
 The sum minus s = the larger number (or other number)

PROBLEMS

- 1) The sum of two numbers is 50. Five times the smaller is 5 more than twice the larger. What are the numbers?

Let $s =$ the smaller part (number) } outline
 $50 - s =$ the larger part (number)

$$5s = 2(50 - s) + 5$$

$$5s = 100 - 2s + 5$$

$$5s + 2s = 100 + 5$$

$$7s = 105$$

$s = 15$ The smaller

$50 - s = 35$ The larger

- 2) Separate the number 32 into two parts such that 5 times the larger is 9 less than eight times the smaller.

Let $s =$ the smaller part

$32 - s =$ the larger part

$$5(32 - s) = 8s - 9$$

$$160 - 5s = 8s - 9$$

$$+5s + 8s = +9 + 160$$

$$13s = 169$$

$s = 13$ The smaller part

$32 - s = 19$ The larger part

Problems on the use of the word "exceeds"

KEEP IN MIND

I. Exceeds means "is more than".

If "this" exceeds "that" by "so much" then
 "this" = "that" + "so much"

II. Since 8 exceeds 5 by 3 then

$$8 = 5 + 3$$

III. If we know the difference between two numbers we - Let s = the smaller number
 the difference + s = the larger number

PROBLEMS

- 1) Separate the number 20 into two parts such that 4 times the larger exceeds 3 times the smaller by 17

Let s = the smaller part } outline
 $20 - s$ = the larger part }

$$4(20 - s) = 3s + 17$$

$$80 - 4s = 3s + 17$$

$$+7s = +63$$

$s = 9$ The smaller part

$20 - s = 11$ The larger part

- 2) The larger of two numbers is 3 more than 5 times the smaller. The larger exceeds 5 times the smaller by one less than the smaller. Find the numbers.

Let s = the smaller

$5s + 3$ = the larger

$$5s + 3 = 5s + s - 1$$

$+s = +4$ the smaller

$5s + 3 = 23$ the larger

Coin problems

KEEP IN MIND

- I. Set up an outline about each kind of coin.
- II. Express the value of each kind of coin on the right side of the vertical line in the value column.
- III. Set up your equation as follows: The value of one kind of coin plus the value of all the other kinds of coins = the total amount of money that is given in the problem.
- IV. Your answer cannot be fractional or negative. If it is, the problem has no solution.

PROBLEMS

- 1) Donald has \$4.85 in nickels, dimes, and pennies. There are three times as many pennies as dimes, and 5 more nickels than twice the number of dimes. Find the number of each kind of coin.

VALUE

Let d = the no. of dimes - $10d$ cents

$3d$ = the no. of pennies - $1(3d)$ cents

$2d + 5$ = the no. of nickels - $5(2d + 5)$ cents

$$10d + 1(3d) + 5(2d + 5) = 485$$

$$10d + 3d + 10d + 25 = 485$$

$$23d = 460$$

$$d = 20 \text{ Dimes}$$

$$3d = 60 \text{ pennies}$$

$$2d + 5 = 45 \text{ nickels}$$

Age Problems

PROBLEMS

- 1) A boy is 10 years older than his brother. Four years from now the boy will be twice as old as his brother is then. Find the age of each now.

Let x = brother's age now

$x + 10$ = boy's age now

$$x + 10 + 4 = 2(x + 4)$$

$$x + 10 + 4 = 2x + 8$$

$+x = +6$ brother's age now

$x + 10 = 16$ boy's age now

- 2) A father is 4 times as old as his son. In 20 years the father's age will be twice the son's age then. How old is each now?

Let s = son's age now

$4s$ = father's age now

$$4s + 20 = 2(s + 20)$$

$$4s + 20 = 2s + 40$$

$$2s = 20$$

$s = 10$ - son's age now

$4s = 40$ father's age now

3) The sum of the ages of John and Mary is 32 years. Four years ago John was twice as old as Mary was then. How old is each now?

$$\begin{aligned}
\text{Let } m &= \text{Mary's age now} \\
32 - m &= \text{John's age now} \\
32 - m - 4 &= 2(m - 4) \\
32 - m - 4 &= 2m - 8 \\
-3m &= -36 \\
m &= 12 \text{ Mary's age now} \\
32 - m &= 20 \text{ John's age now}
\end{aligned}$$

4) Ten years from now a man will be twice as old as he was 10 years ago. How old is he now?

$$\begin{aligned}
\text{Let } a &= \text{man's age now} \\
a + 10 &= 2(a - 10) \\
a + 10 &= 2a - 20 \\
+ a &= + 30 \text{ man's age now}
\end{aligned}$$

5) A man is 32 years old and his son is 8 years old. In how many years will the father be 3 times as old as his son.

$$\begin{aligned}
\text{Let } n &= \text{the no. of years} \\
32 + n &= 3(8 + n) \\
32 + n &= 24 + 3n \\
n &= 4 \text{ no. of years}
\end{aligned}$$

Consecutive integer problems

KEEP IN MIND

I An integer is a positive or negative whole number.

II. The outline for consecutive integers is:

Let $x =$ the first

$x+1 =$ the second

$x+2 =$ the third

III. The outline for consecutive odd or even integers is:

Let $x =$ the first

$x+2 =$ the second

$x+4 =$ the third

IV. Your answer cannot be a fraction because an integer is a whole number.

PROBLEMS

- 1) Find three consecutive even integers whose sum is 150.

Let $x =$ the first

$x+2 =$ the second

$x+4 =$ the third

$$x + x + 2 + x + 4 = 150$$

$$3x = 144$$

$$x = 48 \text{ the first}$$

$$x + 2 = 50 \text{ the second}$$

$$x + 4 = 52 \text{ the third}$$

- 2) Find three consecutive integers such that 5 times the largest decreased by the second exceeds three times the smallest by 18.

$$\text{Let } x = \text{first (smallest)}$$

$$x + 1 = \text{second}$$

$$x + 2 = \text{third (largest)}$$

$$5(x + 2) - (x + 1) = 3x + 18$$

$$5x + 10 - x - 1 = 3x + 18$$

$$5x - x - 3x = 18 - 10 + 1$$

$$x = 9 \text{ smallest}$$

$$x + 1 = 10 \text{ second}$$

$$x + 2 = 11 \text{ largest}$$

- 3) Find four consecutive odd integers such that 4 times the sum of the second and third increased by the fourth, is 4 less than 11 times the first.

$$\text{Let } x = \text{the first}$$

$$x + 2 = \text{the second}$$

$$x + 4 = \text{the third}$$

$$x + 6 = \text{the fourth}$$

$$4(2x+6) + (x+6) = 11x - 4$$

$$8x + 24 + x + 6 = 11x - 4$$

$$8x + x - 11x = -4 - 6 - 24$$

$$-2x = -34$$

$$x = 17 \text{ First}$$

$$x + 2 = 19 \text{ Second}$$

$$x + 4 = 21 \text{ Third}$$

$$x + 6 = 23 \text{ Fourth}$$

- 4) Is it possible to have three consecutive odd integers such that twice the third increased by three times the first is 17 less than 7 times the second?

Let x = the first

$x + 2$ = the second

$x + 4$ = the third

$$2(x+4) + 3x = 7(x+2) - 17$$

$$2x + 8 + 3x = 7x + 14 - 17$$

$$2x + 3x - 7x = -8 + 14 - 17$$

$$-2x = -11$$

$$x = 5\frac{1}{2} \quad \text{NOT POSSIBLE}$$

Perimeter problems

KEEP IN MIND

I. The sum of all the sides of any figure is its perimeter.

II. Your answer may be fractional but not negative. If it is negative, the problem has no solution.

III. $2(\text{length}) + 2(\text{width}) = \text{the perimeter of a rectangle.}$

PROBLEMS

- 1) The perimeter of a certain triangle is 44 inches. The second side is one inch less than twice the first side. The third side is 5 inches more than twice the first side. Find the length of each side.

Let $f = \text{first side in inches}$

$2f - 1 = \text{second side in inches}$

$2f + 5 = \text{third side " "}$

$$f + 2f - 1 + 2f + 5 = 44$$

$$5f = 40$$

$f = 8$ first side

$2f - 1 = 15$ second side

$2f + 5 = 21$ third side

- 2) The length of a certain rectangle is 4 inches less than 5 times the width. The perimeter of the rectangle is one inch more than 9 times the width. Find the length and the width.

Let w = the no. of inches in the width

$5w - 4$ = the no. " " " " length

$$2(w) + 2(5w - 4) = 9w + 1$$

$$2w + 10w - 8 = 9w + 1$$

$$2w + 10w - 9w = 1 + 8$$

$$3w = 9$$

$$w = 3 \text{ Width}$$

$$5w - 4 = 11 \text{ Length}$$

Problems on, "The sum of all the parts of anything = the whole thing"

KEEP IN MIND

Let ψ = the whole thing and set it equal to the sum of all its parts.

PROBLEMS

- 1) A man spent $\frac{1}{3}$ of his money and 6 dollars more, and found that he had 8 dollars more than half of it left. How many dollars did he have at first?

Let x = the number of dollars

$$\frac{1}{3}x + 6 + \frac{1}{2}x + 8 = x$$

$$2x + 36 + 3x + 48 = 6x$$

$$+x = +84 \text{ The number of dollars}$$

- 2) The age of Diophantus, a brilliant Greek mathematician, may be found from the following information: Diophantus spent one sixth of his life in childhood, one twelfth in youth, and one seventh as a bachelor; five years after his marriage a son was born who died 4 years before his father at half his father's age at death. How old was Diophantus when he died?

Let a = his age in years

$$M_{84} \quad \frac{1}{6}a + \frac{1}{12}a + \frac{1}{7}a + 5 + \frac{1}{2}a + 4 = a$$

$$14a + 7a + 12a + 420 + 42a + 336 = 84a$$

$$14a + 7a + 12a + 42a - 84a = -336 - 420$$

$$+9a = +756$$

$$a = 84 \text{ his age in years}$$

Mixture problem

KEEP IN MIND

In solving a mixture problem we frequently break up a number into two parts and set up an equation of value. The value of one part plus the value of the other part = the value of the mixture

PROBLEMS

- 1) A grocer has two kinds of coffee, one worth 60¢ a lb. and the other 80¢ a lb. He wishes to make a mixture of 100 pounds to sell at 76¢ a lb. How many pounds of each must he use in the mixture?

	VALUE
Let s = no. of lbs. at 60¢ a lb.	60 s cents
$100 - s$ = " " " " 80¢ " "	80(100 - s) cents
100 = " " " " 76¢ " "	76(100) cents

$$60s + 80(100 - s) = 76(100)$$

$$60s + 8000 - 80s = 7600$$

$$+ 20s = +400$$

$$s = 20 \text{ no. of lbs. at } 60¢ \text{ a lb.}$$

$$100 - s = 80 \text{ no. " " " } 80¢ \text{ " "}$$

- 2) How many pounds of tea at 32¢ a lb. must be added to 36 lbs. of tea at 50¢ a lb. to make a mixture of tea that can be sold at 40¢ a lb.

	VALUE
Let $t =$ no. of lbs. at 32¢ a lb.	$32t$ cents
(26 lbs) $36 =$ " " " " 50¢ " "	$50(36)$ cents
$t+36 =$ " " " " 40¢ " "	$40(t+36)$ cents

$$32t + 50(36) = 40(t+36)$$

$$32t + 1800 = 40t + 1440$$

$$-8t = -360$$

$$t = 45 \text{ no. of lbs. at } 32 \text{¢ a lb.}$$

- 3) How many pounds of candy worth "a" cents a lb. and of other candy worth "b" cents a pound must be mixed to make 100 pounds worth "c" cents a lb.?

	VALUE
Let $x =$ no. of lbs. at "a"¢ a lb.	ax cents
$100-x =$ " " " " "b"¢ " "	$b(100-x)$ cents
$100 =$ " " " " "c"¢ " "	$100c$ cents

$$ax + b(100-x) = 100c$$

$$ax + 100b - xb = 100c$$

$$ax - bx = 100c - 100b$$

$$x(a-b) = 100(c-b)$$

$$x = \frac{100(c-b)}{a-b} = \text{no. of lbs. at "a"¢ a lb.}$$

$$100-x = \frac{100(a-c)}{a-b} = \text{no. of lbs. at "b"¢ a lb.}$$

Problems on finding the selling price

KEEP IN MIND:

- I. Let s = the selling price
 II. Selling price = cost + margin (profit)

PROBLEMS

- 1) A hat cost a milliner \$2.80. For how much must it be sold in order to have a margin of 60% of the selling price?

$$\begin{aligned} \text{Let } s &= \text{selling price in } \$ \\ s &= 2.80 + .60s \\ \{ \text{M}_{100} \quad 100s &= 280 + 60s \} \\ 40s &= 280 \\ s &= 7 \text{ selling price in } \$ \end{aligned}$$

- 2) A suit of clothes cost a merchant \$22. For how much must he sell it to get 35% of the selling price for his overhead and 10% of it for profit?

$$\begin{aligned} \text{Let } s &= \text{selling price in } \$ \\ s &= 22 + .35s + .10s \\ \text{M}_{100} \quad 100s &= 2200 + 35s + 10s \\ 55s &= 2200 \\ s &= 40 = \text{selling price in } \$ \end{aligned}$$

Problems about the terms of a fraction

KEEP IN MIND

I. $\frac{\text{numerator}}{\text{denominator}} = \text{Fraction}$

PROBLEMS

- 1) The denominator of a fraction is 7 more than the numerator. If the numerator is increased by 2 and the denominator is increased by 1 the resulting fraction has the value $\frac{1}{2}$. Find the fraction.

$$\left. \begin{array}{l} \text{Let } n = \text{numerator} \\ n + 7 = \text{denominator} \end{array} \right\} \text{fraction}$$

$$\frac{n+2}{n+8} = \frac{1}{2}$$

$$2(n+2) = n+8$$

$$2n - n = 8 - 4$$

$$n = 4$$

$$\left. \begin{array}{l} n = 4 \text{ numerator} \\ n + 7 = 11 \text{ denominator} \end{array} \right\} \text{fraction}$$

- 2) What number added to both terms of the fraction $\frac{9}{14}$ will change its value to $\frac{1}{2}$?

Let $n =$ the no. to be added

$$\frac{9+n}{14+n} = \frac{1}{2}$$

M $2(14+n)$

$$2(9+n) = 14+n$$

$$18+2n = 14+n$$

$$n = -4 \text{ no. to be added}$$

Interest problems

KEEP IN MIND

I. The interest or income on one sum of money + the interest or income on the other sums of money = the total interest or income.

II. If you do not know the total interest, compare the interests on the different sums instead.

PROBLEMS

- 1) A man has \$8000. He wishes to invest a part of it at 5% and the other part at $5\frac{1}{2}\%$ so as to have a total yearly interest income of \$425. How much must he invest at each rate?

INTEREST

Let f = no. of \$ at 5%	\$.05 f
$8000 - f$ = " " " " 5½%	\$.055(8000 - f)
8000 = " " " earning 425	\$ 425

$$.05f + .055(8000 - f) = 425$$

$$M_{1000} \quad .05f + 440 - .055f = 425$$

$$50f + 440,000 - 55f = 425,000$$

$$+5f = +15,000$$

$$f = 3,000 \text{ no. of } \$ \text{ at } 5\%$$

$$8000 - f = 5,000 \text{ " " " " } 5\frac{1}{2}\%$$

- 2) If \$ 1,500 is invested at 5% and \$ 2,000 at 6%, how much must be invested at 7% in order to make the total interest income 6% of the total original income?

INTEREST

Let s = no. of \$ at 7%	\$.07 s
1500 = " " " " 5%	\$ 75
2000 = " " " " 6%	\$ 120
$s + 3500$ = total avg. 6%	\$.06($s + 3500$)

$$.07s + 75 + 120 = .06(s + 3500)$$

$$.07s + 75 + 120 = .06s + 210$$

$$M_{100} \quad 7s + 7500 + 12,000 = 6s + 21,000$$

$$s = \$1,500 \text{ no. of } \$ \text{ at } 7\%$$

- 3) Part of \$N is invested at 5% and the remainder at 2%. How much is invested at each rate, if the total income or interest is 4% of the total investment?

	INTEREST
Let $f =$ no. of \$ at 5%	\$.05 f
$N - f =$ " " " " 2%	\$.02 $(N - f)$
$N =$ " " " avg. 4%	\$.04 N

$$.05f + .02(N - f) = .04N$$

M100 $.05f + .02N - .02f = .04N$

$$5f + 2N - 2f = 4N$$

$$3f = 2N$$

$$f = \frac{2N}{3} \text{ no. of \$ at 5\%}$$

$$N - f = \frac{N}{3} \text{ " " " " 2\%}$$

- 4) A man has two investments that total \$6000. The interest on one at 5% is \$24 less than the interest on the other at 4%. How much has he invested at each rate?

	INTEREST
Let $v =$ no. of \$ at 5%	\$.05 v
$6000 - v =$ " " " " 4%	\$.04 $(6000 - v)$

$$.05v = .04(6000 - v) - 24$$

M100 $.05v = 240 - .04v - 24$

$$5v + 4v = 24,000 - 2,400$$

$$v = 2,400 \text{ no. of \$ at 5\%}$$

$$6000 - v = 3,600 \text{ " " " " 4\%}$$

Work and Pipe Problems

KEEP IN MIND

I. The part of the work A can do in one day + the part B can do in one day = the part of work both can do in one day.

II. If n = the no. of days it will take to do the work together $\frac{1}{a} + \frac{1}{b} = \frac{1}{n}$

III. The part of work A can do in the no. of days he works + the part of work B can do in the number of days he works = the part of the work that is complete. If the job is finished this number is 1.

PROBLEMS

1) James can plow a garden in 6 hours alone, and Ben can plow it in 4 hours alone. How long will it take James & Ben to plow it when working together?

M12m

$$\frac{1}{6} + \frac{1}{4} = \frac{1}{n}$$

$$2n + 3n = 12$$

$$5n = 12$$

$$n = 2\frac{2}{5} \text{ hours}$$

- 2) A & B when working together can do a certain piece of work in 2 days. If A alone can do the work in 3 days, how many days will it take B to do the work?

Let B = no. of days for B alone

$$\frac{1}{3} + \frac{1}{B} = \frac{1}{2}$$

M_{6B}

$$2B + 6 = 3B$$

$$B = 6 \text{ no. of days for B alone}$$

- 3) Frank can do a piece of work alone in 8 days. After he has worked 3 days, Walter joins him and they finish the job in 2 more days. How long will it take Walter alone to do the whole job?

Let x = no. of days for Walter alone

$$\frac{3}{8} + \frac{2}{8} + \frac{2}{x} = 1$$

M_{8x}

$$3x + 2x + 16 = 8x$$

$$3x = 16$$

$$x = 5\frac{1}{3}, \text{ no. of days Walter alone}$$

- 4) A swimming tank is fitted with two pipes. One pipe alone can fill the tank in 3 hours and the other pipe alone can fill it in 6 hours. When both pipes are open at the same time and the tank is empty, how long will it take to fill the tank?

Let $n =$ no. of hours to fill tank

$$\frac{1}{3} + \frac{1}{6} = \frac{1}{n}$$

M_{6m} $2n + n = 6$

$n = 2$, no. of hours to fill tank

- 5) A tank can be filled by one pipe in 4 hours and emptied by another in 5 hours. If both pipes are open, and the tank is empty, how long will it take before the tank is filled?

Let $n =$ no. of hours

$$\frac{1}{4} - \frac{1}{5} = \frac{1}{n}$$

M_{20m} $5n - 4n = 20$

$n = 20 =$ no. of hours

Area problems

PROBLEMS

- 1) The length of a certain lot is 15 ft. more than 3 times the width. If the width is increased by 5 ft., and the length by 10 ft., the area is increased by 1,000 ft.². What are the dimensions?

Let w = width in feet

$3w + 15$ = length in "

$$(w + 5)(3w + 25) = w(3w + 15) + 1000$$

$$3w^2 + 40w + 125 = 3w^2 + 15w + 1000$$

$$25w = 875$$

$$w = 35 \text{ width in feet}$$

$$3w + 15 = 120 \text{ length " "}$$

Problems on solutions

PROBLEMS

- 1) How much alcohol must be added to two ⁽²⁾ quarts of a 30% solution of alcohol and water to make a 50% solution?

Let x = no. of qts. to be added

$$.30(2) + x = .50(2+x)$$

M₁₀₀

$$.6 + x = .5x + 1$$

$$6 + 10x = 10 + 5x$$

$$5x = 4$$

$$x = \frac{4}{5} \text{ no. of qts. added}$$

- 2) How much distilled water must be added to a qt. of an antiseptic solution that contains 20% alcohol to make a solution that contains 15% alcohol?

Let x = no. of qts. to be added

$$.20(1) = .15(x+1)$$

M₁₀₀ $.20 = .15x + .15$

$$20 = 15x + 15$$

$$+15x = +5$$

$$x = \frac{1}{3} \text{ no. of qts. to be added}$$

- 3) An automobile radiator contains 18 gts of a 20% solution of H_2O and antifreeze. How much must be replaced by pure antifreeze to make a solution containing 40% of antifreeze?

Let x = no. of gts. to be replaced

$$.20(18) - .20x + x = .40(18)$$

$$M_{10} \quad 3.6 - .2x + x = 7.2$$

$$36 - 2x + 10x = 72$$

$$8x = 36$$

$$x = 4\frac{1}{2}, \text{ no. of gts. replaced}$$

- 4) How much H_2O must be \uparrow from 100 lbs. of a 12% sol. of $NaCl$ in order to obtain a 20% sol.?

Let w = no. of lbs. of H_2O \uparrow

$$.12(100) = .20(100 - w)$$

$$M_{10} \quad 12 = 20 - .2w$$

$$120 = 200 - 2w$$

$$2w = 80$$

$$w = 40 = \text{no. of lbs. of } H_2O \uparrow$$

Time, rate and distance problems

KEEP IN MIND

I. When traveling toward each other or away from each other the distance one travels + the distance the other travels = the distance they are apart.

II. Rate \times time = distance

III. $\frac{\text{Distance}}{\text{Rate}} = \text{time}$

IV. When overtaking someone the distance one travels = the distance the other travels.

V. In a round trip problem the time going + the time returning = the total time.

VI. Sometimes it is possible to find the rate by comparing the times.

VII. Always set up a Time, Rate, Distance box.

VIII. Express the time in hours unless otherwise asked.

PROBLEMS

- 1) Two cars started at the same time 8 A.M., from the same place and traveled in opposite directions. Their rates were 25 and 35 miles per hour respectively. After how many hours will they be 180 miles apart? What time will it be?

	FIRST	SECOND
TIME	x	x
RATE	25	35
DISTANCE	$25x$	$35x$

Let x = the no. of hours each travels.

$$25x + 35x = 180$$

$$60x = 180$$

$x = 3$ no. of hours each travels.

The time will be 11 A.M.

- 2) An automobile party is traveling 15 miles an hour. At what rate must a second party travel in order to overtake the first if the second party starts from the same place 2 hours after the first party left?

	FIRST	SECOND
TIME	4	3
RATE	15	x
DISTANCE	$4(15)$	$3(x)$

Let x = rate of the second party

$$3(x) = 4(15)$$

$$3x = 60$$

$x = 20$ rate of the second party

- 3) One automobile party is traveling 15 miles an hour. In how many hours will a second party traveling 25 miles an hour overtake the first if the second party starts from the same place 2 hours after the first party left?

	FIRST	SECOND
TIME	$2+x$	x
RATE	15	25
DISTANCE	$15(2+x)$	$x(25)$

Let x = no. of hours second party travels

$$15(2+x) = 25x$$

$$30 + 15x = 25x$$

$$+10x = +30$$

$x = 3$ no. of hours second travels

- 4) A stream is flowing 3 miles an hour. A motor boat on it is exerting a force that would carry it 12 miles per hour. How far downstream can the boat go and return if it makes the round trip in 4 hours?

	DOWN- STREAM	UP- STREAM
DISTANCE	n	n
RATE	15	9
TIME	$\frac{n}{15}$	$\frac{n}{9}$

Let n = no. of miles it can go downstream and back

$$\text{H45} \quad \frac{n}{15} + \frac{n}{9} = 4$$

$$3n + 5n = 180$$

$$8n = 180$$

$$n = 22\frac{1}{2} \text{ - no. of miles}$$

- 5) An airplane that travels 3 times as fast as an automobile takes ^{6 hours} less time than the auto to go 270 miles. What is the average rate?

	AUTO	PLANE
DISTANCE	270	270
RATE	x	$3x$
TIME	$\frac{270}{x}$	$\frac{270}{3x}$

Let x = rate of the auto in m.p.h.

$3x$ = " " " plane " "

$$M_{3x} \quad \frac{270}{3x} = \frac{270}{x} - \frac{6}{1}$$

$$270 = 810 - 18x$$

$$18x = 540$$

$x = 30$ - rate of auto in m.p.h.

$3x = 90$ - " " plane " " "

- 6) The rate of a passenger train exceeds that of a freight train by 22 m.p.h. The passenger train goes 252 miles in the same time that the freight train goes 120 miles. Find the rate of each train.

	PASSENGER	FREIGHT
DISTANCE	252	120
RATE	$f + 22$	f
TIME	$\frac{252}{f+22}$	$\frac{120}{f}$

Let f = rate of freighter in m.p.h.

$f + 22$ = " " passenger " " "

$$M \quad f(f+22) \quad \frac{252}{f+22} = \frac{120}{f}$$

$$252f = 120(f+22)$$

$$252f = 120f + 2640$$

$$252f - 120f = 2640$$

$f = 20$ freight's rate

$f + 22 = 42$ passenger's rate

Number relation problems

KEEP IN MIND

I. If a number is broken up into two parts or we know the sum of two numbers we set up our equation as follows:

The larger part + the smaller part = the number
 $l + s = \text{the number}$

II. We set up another equation from another statement in the problem.

III. If the problem requires a division then:
 $\frac{\text{no. divided}}{\text{divisor}} = \text{Quotient} + \frac{\text{remainder}}{\text{divisor}}$

PROBLEMS

1) Separate 65 into two parts such that the larger is 2 more than twice the smaller.

Let $l = \text{larger}$

$s = \text{smaller}$

I $l + s = 65$

II $l = 2s + 2$

I $2s + 2 + s = 65$

$3s = 63$

$s = 21$ smaller

$l = 44$ larger

- 2) The sum of two numbers is 28. Their difference is six. Find the numbers.

Let l = larger
 s = smaller

$$I \quad l + s = 28$$

$$II \quad l - s = 6$$

$$l = 6 + s$$

$$I \quad 6 + s + s = 28$$

$$2s = 22$$

$$s = 11 \text{ smaller}$$

$$l = 17 \text{ larger}$$

- 3) Find two numbers such that 3 times the first exceeds twice the second by 9, and 4 times the second is 51 less than 7 times the first.

Let f = first
 s = second

$$I \quad 3f = 2s + 9$$

$$I \quad 3f - 2s = 9$$

$$II \quad 4s = 7f - 51$$

$$II \quad 4s - 7f = -51$$

$$H_2 \quad I \quad -4s + 6f = 18$$

$$+ f = +33 \text{ first}$$

$$4s = 231 - 51$$

$$4s = 180$$

$$s = 45 \text{ second}$$

- 4) The sum of 2 numbers is 84. The larger is 4 more than 3 times the smaller. Find the numbers.

Let l = larger

s = smaller

$$I \quad l + s = 84$$

$$II \quad l = 3s + 4$$

$$I \quad 3s + 4 + s = 84$$

$$4s = 80$$

$$s = 20 \text{ smaller}$$

$$l = 64 \text{ larger}$$

- 5) The sum of 2 numbers is 73. If the larger is divided by the smaller the quotient is 2 and the remainder is 16. Find the numbers.

Let l = larger

s = smaller

$$I \quad l + s = 73$$

$$II \quad \frac{l}{s} = 2 + \frac{16}{s}$$

$$I \& II \quad l = 2s + 16$$

$$I \quad 2s + 16 + s = 73$$

$$3s = 57$$

$$s = 19 \text{ smaller}$$

$$l = 54 \text{ large}$$

6) The difference of 2 numbers is 8. The quotient obtained by dividing the larger by the smaller is 3 and the remainder is 2. What are the numbers?

Let l = larger number

s = smaller

$$I \quad l - s = 8$$

$$II \quad \frac{l}{s} = 3 + \frac{2}{s}$$

$$M \times II \quad l = 3s + 2$$

$$I \quad 3s + 2 - s = 8$$

$$2s = 6$$

$$s = 3 \text{ smaller}$$

$$l = 11 \text{ larger}$$

Problems about two digit numbers

KEEP IN MIND

I. The outline (which must be memorized) to solve these problems is as follows:

Let t = tens' digit

u = units' digit

$10t + u$ = number (old no. or original no.)

$10u + t$ = number reversed (new number)

$t + u$ = sum of the digits

II. If " t " or " u " is fractional or negative.

NO NUMBER EXISTS

PROBLEMS

- 1) The sum of the digits of a certain two digit number is 6. If the digits are reversed, the new number is 36 more than the old number. What is the number?

Let t = tens' digit

u = units' digit

$10t + u$ = the number

$$I \quad t + u = 6$$

$$u = 6 - t$$

$$II \quad 10u + t = 10t + u + 36$$

$$D_9 \quad 9u - 9t = 36$$

$$II \quad u - t = 4$$

$$6 - t - t = 4$$

$$-2t = -2$$

$$t = 1$$

$$u = 5$$

$(10t + u) = 15$ the number

- 2) If a certain number of two digits is divided by the sum of its digits, the quotient is 4 and the remainder is 3. If the digits are reversed, the sum of the resulting number and 23 is twice the original number. Find the number.

Let $t =$ tens digit

$u =$ units' digit

$10t + u =$ the number

$$I \quad \frac{10t + u}{t + u} = 4 + \frac{3}{t + u}$$

$$7_{t+u} \quad 10t + u = 4t + 4u + 3$$

$$6t - 3u = 3$$

$$D_3 \quad 2t - u = 1$$

$$u = 2t - 1$$

$$\text{II } 10u + t + 23 = 2(10t + u)$$

$$10u + t + 23 = 20t + 2u$$

$$8u - 19t = -23$$

$$8(2t - 1) - 19t = -23$$

$$16t - 8 - 19t = -23$$

$$-3t = -15$$

$$t = 5$$

$$u = 9$$

$$10t + u = 59 = \text{the number}$$

- 3) Find a number of two digits such that the sum of it and twice the number made by reversing the digits is 207, and twice the tens' digit exceeds the units' digit by three.

Let $t =$ tens' digit

$u =$ units' digit

$10t + u =$ the number

$$\text{I } 10t + u + 2(10u + t) = 207$$

$$10t + u + 20u + 2t = 207$$

$$12t + 21u = 207$$

$$D_3 \quad 4t + 7u = 69$$

$$\text{II } 2t = u + 3$$

$$+u = +2t - 3$$

$$4t + 7(2t - 3) = 69$$

$$4t + 14t - 21 = 69$$

$$18t = 90$$

$$t = 5 \quad u = 7 \quad 10t + u = 57 \text{ number}$$

- 4) If the digits of a certain two digit no. are reversed, the new number plus twice the original number is 168. The number exceeds four times the sum of its digits by three. Find the number.

t = tens' digit

u = units' digit

$10t + u$ = the number

$$I \quad 10u + t + 2(10t + u) = 168$$

$$10u + t + 20t + 2u = 168$$

$$21t + 12u = 168$$

$$D_3 \quad 7t + 4u = 56$$

$$II \quad 10t + u = 4(t + u) + 3$$

$$10t + u = 4t + 4u + 3$$

$$6t - 3u = 3$$

$$D_3 \quad 2t - u = 1$$

$$u = 2t - 1$$

$$I \quad 4(2t - 1) + 7t = 56$$

$$8t - 4 + 7t = 56$$

$$15t = 60$$

$$t = 4$$

$$u = 7$$

$$10t + u = 47 \text{ - The number}$$

Perimeter Problems

PROBLEMS

- 1) The length of a lot is 30 feet more than twice the width. The perimeter is 300 feet. Find the dimensions of the lot.

Let l = length in feet
 w = width " "

$$I \quad l = 2w + 30$$

$$II \quad 2l + 2w = 300$$

$$D_2 \quad l + w = 150$$

Mixture problems

PROBLEMS

- 1) A grocer has two kinds of coffee, one worth 60¢ a pound and the other 80¢ a pound. He wishes to make a mixture of 100 pounds to sell at 76 cents a pound. How many pounds of each must he use in the mixture?

	VALUE
Let x = the no. of lbs at 60¢ a lb.	$60x$ cents
e = " " " " " 80¢ " "	$80e$ cents
100 = " " " " " 76¢ " "	$76(100)$ cents

$$I \quad s + e = 100$$

$$II \quad 60s + 80e = 7600$$

$$D_{20} \quad 3s + 4e = 380$$

- 2) The admission tickets for an entertainment were 25¢ each for adults and 10¢ each for children. If 385 persons entered and the gate receipts were \$62.65, how many children and how many adults entered?

Let c = the no. of children at 10¢ each	VALUE 10c cents
a = " " " adults " 25¢ "	25a cents
385 = " " " persons paying	62.65 cents

$$I \quad c + a = 385$$

$$II \quad 10c + 25a = 6265$$

$$D_5 \quad 2c + 5a = 1253$$

- 3) A purse has \$4.70 in dimes and quarters. There are 29 coins in all. How many of each kind are there?

Let d = the no. of dimes	VALUE 10d cents
q = " " " quarters	25q cents
29 = the total no. of coins	4.70

$$I \quad d + q = 29$$

$$II \quad 10d + 25q = 470$$

$$D_5 \quad 2d + 5q = 94$$

- 4) Corn has about 9% protein in it. Soybean meal has about 16%. How many lbs. of each should be taken to make a 100 lb. mixture that has 14% protein?

	PURE PROTEIN
Let c = no. of lbs. of corn with 9% pro.	$.09c$ lbs.
s = " " " " soybean " 16% "	$.16s$ lbs.
100 = " " " " both " 14% "	$.14(100)$ lbs.

$$I \quad c + s = 100$$

$$II \quad .09c + .16s = .14(100)$$

$$M_{100} \quad 9c + 16s = 1400$$

- 5) A workman and his helper received \$31⁰⁰ for 20 hrs. work. The workman received 40¢ an hour less than twice the pay of the helper for an hour. How much per hr. did each receive?

Let w = no. of cents per hr. workman received

h = " " " " helper " "

$$I \quad 20w + 20h = 3100$$

$$D_{20} \quad w + h = 155$$

$$II \quad w = 2h - 40$$

- 6) A telegram of 18 wds. costs 69¢, and one of 25 words sent to the same address costs 90¢. If the charge consists of a flat rate for the first ten wds. and an extra charge for each word over ten, what is the flat rate and the extra charge per wd.

Let f = the flat rate for first 10 words in cents
 e = the extra charge per word over 10 " "

$$I \quad f + 8e = 69¢$$

$$II \quad f + 15e = 90¢$$

- 7) A customer bought 3 cans of corn and 5 cans of tomatoes for \$1.45. The next customer bought 2 cans of corn and 3 cans of tomatoes for 90¢. Find the cost of one can of each.

Let c = cost of a can of corn in cents

t = " " " " "toms" "

$$I \quad 3c + 5t = 145$$

$$II \quad 2c + 3t = 90$$

Work and pipe problems

PROBLEMS

- 1) Miss Davis and Miss Evans together do 100 pages of typing in 4 hours. At another time, they did the same amount of work when Miss Davis worked 5 hours and Miss Evans worked 2 hours. How long would it take each of the typists to do the work alone?

Let d = no. of hrs. for Davis alone

e = " " " " Evans "

$$I \quad \frac{1}{d} + \frac{1}{e} = \frac{1}{4}$$

$$II \quad \frac{5}{d} + \frac{2}{e} = \frac{1}{1}$$

$$M_2 \quad I \quad \frac{2}{d} + \frac{2}{e} = \frac{2}{4}$$

$$M_4d \quad \frac{3}{d} = \frac{2}{4}$$

$$12 = 2d$$

$d = 6$ no. of hrs. for Davis alone

$$I \quad \frac{1}{6} + \frac{1}{e} = \frac{1}{4}$$

$$2e + 12 = 3e$$

$$2e - 3e = -12$$

$e = 12$ no. of hrs. for Evans alone

2) A swimming pool was filled by a pipe and fire hose together in six hours. At another time it was filled by using a pipe 8 hrs. and the fire hose 3 hours. How long would it take to fill the pool using either the pipe alone or the fire hose alone?

Let p = no. of hrs. for pipe alone

h = " " " " hose "

$$I \quad \frac{1}{p} + \frac{1}{h} = \frac{1}{6}$$

$$II \quad \frac{8}{p} + \frac{3}{h} = 1$$

$$M_3 \quad I \quad \frac{3}{p} + \frac{3}{h} = \frac{3}{6}$$

$$\frac{5}{p} = \frac{3}{6}$$

$$M_6 \quad 30 = 3p$$

$p = 10$ - no. of hrs. for pipe alone

$$\frac{1}{10} + \frac{1}{h} = \frac{1}{6}$$

$$30h \quad 3h + 30 = 5h$$

$$3h - 5h = -30$$

$$-2h = -30$$

$h = 15$ - no. of hrs. for hose alone

Investment problems

PROBLEMS

- 1) A man invested \$20,000, part at 2% interest and the other part at 3% interest. The income (interest) yielded each year was \$400. How much was invested at each rate?

	INTEREST
Let x = the no. of dollars at 2%	\$.02 x
e = " " " " " 3%	\$.03 e
20,000 = " " " " altogether	\$440

$$I \quad x + e = 20,000$$

$$x = 20,000 - e$$

$$II \quad .02x + .03e = 440$$

$$M_{100} \quad 2x + 3e = 44,000$$

$$2(20,000 - e) + 3e = 44,000$$

$$40,000 - 2e + 3e = 44,000$$

$$e = 4,000 - \text{no. of \$ at 3\%}$$

$$x = 16,000 - \text{" " " " 2\%}$$

- 2) A man has 2 investments that total \$6,000. The interest on 1 investment at 5% is \$24 less than the interest on the other investment at 4%. How much has he invested at each rate?

	INTEREST
Let v = no. of \$ at 5%	\$.05 v
f = " " " " 4%	\$.04 f
6000 = " " " altogether	_____

I $v + f = 6000$
 II $.05v = .04f - 24$
 $5v = 4f - 2,400$

- 3) A man has one sum of money invested at 5%, and a second sum \$1,500 larger than the first invested at 7%. His total interest income from these sums is \$321. How much has he invested at each rate?

	INTEREST
Let f = no. of \$ at 5%	\$.05 f
s = " " " " 7%	\$.07 s

I $s = f + 1500$
 II $.05f + .07s = 321$
 $5f + 7s = 32,100$

Age problems

PROBLEMS

- 1) Six years from now John will be twice as old as Charles will be then. Six years ago John was four times as old as Charles was then. How old is each now?

Let J = John's age now in years
 C = Charles' " " " "

$$I \quad J + 6 = 2(C + 6)$$

$$J + 6 = 2C + 12$$

$$J = 2C + 6$$

$$II \quad J - 6 = 4(C - 6)$$

$$J - 6 = 4C - 24$$

$$2C + 6 - 6 = 4C - 24$$

$$-2C = -24$$

$C = 12$ Charles' age now in years

$J = 30$ John's " " " "

- 2) Mary is 4 times as old as Jane. Six years ago Mary was twice as old as Jane will be 3 years from now. How old is each now?

Let m = Mary's age now in years

j = Jane's " " " "

$$I \quad m = 4j$$

$$II \quad m - 6 = 2(j + 3)$$

- 3) Eight years ago John was twice as old as Helen was then. Four years from now, John will be $\frac{5}{4}$ as old as Helen is then. How old is each now?

Let j = John's age now in years

h = Helen's " " " "

$$I \quad j - 8 = 2(h - 8)$$

$$II \quad j + 4 = \frac{5}{4}(h + 4) \quad \left(\frac{5(h+4)}{4} \right)$$

Problems about the terms of a fraction

PROBLEMS

- 1) If 3 is added to both numerator and denominator of a certain fraction its value becomes $\frac{7}{9}$. If 3 is subtracted from both numerator and denominator its value becomes $\frac{1}{3}$. Find the fraction.

Let n = numerator

d = denominator

$\frac{n}{d}$ = the fraction

$$\text{I} \quad \frac{n+3}{d+3} = \frac{7}{9}$$

$$\text{M}(d+3) \quad 9n+27 = 7d+21$$

$$9n - 7d = -6$$

$$\text{II} \quad \frac{n-3}{d-3} = \frac{1}{3}$$

$$\text{M}(d-3) \quad 3n-9 = d-3$$

$$\pm d = -3n + 6$$

$$\boxed{d = 3n - 6}$$

$$\text{I} \quad 9n - 7(3n-6) = -6$$

$$9n - 21n + 42 = -6$$

$$-12n = -48$$

$$n = 4$$

$$d = 6$$

$$\frac{n}{d} = \frac{4}{6} \text{ The fraction (do not reduce)}$$

- 2) The sum of the reciprocals of 2 nos. is $\frac{5}{12}$. Twice the reciprocal of the larger increased by 3 times the reciprocal of the smaller is $\frac{13}{12}$. What are the numbers?

Let l = larger

s = smaller

$$\text{I} \quad \frac{1}{l} + \frac{1}{s} = \frac{5}{12}$$

$$\text{II} \quad \frac{2}{l} + \frac{3}{s} = \frac{13}{12}$$

$$\text{M}_2 \text{ I} \quad \frac{2}{l} + \frac{2}{s} = \frac{10}{12}$$

$$\frac{1}{s} = \frac{3}{12}$$

$3s = 12$ - the smaller

$$s = 4$$

$$\text{I} \quad \frac{1}{l} + \frac{1}{4} = \frac{5}{12}$$

$$\text{M}_{122} \quad 12 + 3l = 5l$$

$$3l - 5l = -12$$

$$-2l = -12$$

$l = 6$ - the larger

Parallel and opposing force problems

KEEP IN MIND

- I $\frac{\text{Distance downstream}}{\text{Rate downstream}} = \text{Time downstream}$
 II $\frac{\text{Distance upstream}}{\text{Rate upstream}} = \text{Time upstream}$
 III $\frac{\text{Distance going}}{\text{Rate going}} + \frac{\text{Distance returning}}{\text{Rate returning}} = \text{TOTAL TIME}$

PROBLEMS

- 1) A motor boat took $1\frac{1}{3}$ hrs. to go 20 miles downstream, but it took $2\frac{2}{9}$ hrs. to come back the same distance upstream. What was its rate in still water and what was the rate of the current?

Let r = rate in still water in mph

c = " of current. " "

I $\frac{20}{r+c} = \frac{4}{3}$

M_{3(r+c)} $60 = 4r + 4c$

D₄ $r+c = 15$

$r = 15 - c$

II $\frac{20}{r-c} = \frac{20}{9}$

M_{9(r-c)} $180 = 20r - 20c$

D₂₀ $c - r = -9$

$c - 15 + c = -9$

$2c = 6$

$c = 3$ rate of current in mph

$r = 12$ " " boat " "

2) An aeroplane traveled a distance of 720 miles in 4 hours when traveling with the wind and 5 hours when traveling against the wind. Find the rate of the wind and the rate of the plane in still air.

Let r = its rate in still air in mph
 w = the " of the wind " " "

$$I \frac{720}{r+w} = 4$$

$$720 = 4r + 4w$$

$$D_4 \quad r + w = 180$$

$$\boxed{r = 180 - w}$$

$$II \frac{720}{r-w} = 5$$

$$720 = 5r - 5w$$

$$D_5 \quad r - w = 144$$

$$180 - w - w = -36$$

$$-2w = -36$$

$w = 18$ rate of wind

$180 - 18 = 162$ " in still air

- 3) A stream is flowing 3 miles an hr. A motor boat that can travel 12 mph in still water makes a trip downstream and back in four (4) hours. How far downstream does the boat go?

Let d = dis. downstream in miles

$$I \quad \frac{d}{15} + \frac{d}{9} = 4$$

$$174 \quad 3d + 5d = 180$$

$$8d = 180$$

$$d = 2.5 \text{ dis. downstream in miles}$$

- Problems that give three first degree equations with three unknowns

PROBLEMS

- 1) The sum of the digits of a number having three digits is 11. If the number is divided by the sum of its hundreds' and units' digits, the quotient is 20 and the remainder is six. The units' digit exceeds the sum of the hundreds' and tens' digits by one. What is the number?

Let h = hundreds' digit

t = tens' "

u = units' "

$$100h + 10t + u = \text{the number}$$

$$\text{I } h + t + u = 11$$

$$\text{II } \frac{100h + 10t + u}{h + u} = 20 + \frac{6}{h + u}$$

$$\text{III } u = h + t + 1$$

- 2) The perimeter of a Δ is 75 inches. The first side exceeds the second side by 10 inches, and it exceeds the third side by 5 inches. How long is each side of the Δ ?

Let f = first side

s = second "

t = third "

$$I \quad f + s + t = 75$$

$$II \quad f = s + 10$$

$$III \quad f = t + 5$$

- 3) In a Δ $\angle A$ exceeds $\angle B$ by 15° , and the sum of $\angle A$ and twice $\angle C$ is 150° . How large is each \angle of the Δ ?

Let $A =$ no. of degrees in $\angle A$

$B =$ " " " " $\angle B$

$C =$ " " " " $\angle C$

$$I \quad A = B + 15$$

$$II \quad A + 2C = 150$$

$$III \quad A + B + C = 180$$

- 4) A and B can do a piece of work in 10 days. B and C can do it in 20 days. A and C can do it in 12 days. How long would it take each of them to do the work alone?

Let $A =$ no. of days for A alone

$B =$ " " " " B "

$C =$ " " " " C "

$$I \quad \frac{1}{A} + \frac{1}{B} = \frac{1}{10}$$

$$II \quad \frac{1}{B} + \frac{1}{C} = \frac{1}{20}$$

$$III \quad \frac{1}{A} + \frac{1}{C} = \frac{1}{12}$$

- 5) The sum of the 3 digits of a number is 23. The digit in the tens' place exceeds that in the units' place by 3. If 198 is subtracted from the given number, the result is the no. formed by reversing the digits of the given number. What is the number?

Let h = hundreds' digit

t = tens' "

u = units' "

$$100h + 10t + u = \text{the number}$$

$$\text{I } h + t + u = 23$$

$$\text{II } t = u + 3$$

$$\text{III } 100h + 10t + u - 198 = 100u + 10t + h$$

- 6) The sum of two of three certain nos. is 5 more than twice the third. If twice the first is increased by the third, the sum is 1 less than the second. Three times the sum of the first and third is 4 less than twice the second. What are the numbers?

Let f = first, s = second, t = third

$$\text{I } f + s = 2t + 5$$

$$\text{II } 2f + t = s - 1$$

$$\text{III } 3(f + t) = 2s - 4$$

The first part of the paper is devoted to a general discussion of the problem. It is shown that the problem is well-posed in the sense of Hadamard. The second part is devoted to the construction of the solution. The third part is devoted to the numerical solution of the problem. The fourth part is devoted to the numerical solution of the problem. The fifth part is devoted to the numerical solution of the problem.

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For a ring of 100 particles of which 50 are inside and 50 are outside, we have to take into account the forces from the particles outside. The force from the particles inside is zero because they are symmetrically distributed.

	INSIDE	OUTSIDE
DISTANCE	200	40
NO.	50	50
FORCE	$\frac{200}{20}$	$\frac{40}{20}$

Force from outside is 20

Force from inside is 0

Net force is 20

Force from outside is 20

Force from inside is 0

Net force is 20

Force from outside is 20

Force from inside is 0

Net force is 20

Force from outside is 20

Force from inside is 0

Net force is 20

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Third section of faint, illegible handwriting, continuing the notes or list.

Fourth section of faint, illegible handwriting, possibly a conclusion or summary.

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I.

For a trip of 260 miles of which 60 miles is inside city limits, a man plans to take 7 hrs. If he expects to drive 30 m.p.h. faster outside city limits than he does inside, what rate must he average outside city limits?

	OUTSIDE	INSIDE
DISTANCE	200	60
RATE	r	$r - 30$
TIME	$\frac{200}{r}$	$\frac{60}{r-30}$

Let r = rate outside in mph

$r - 30$ = " inside " "

$$\frac{200}{r} + \frac{60}{r-30} = \frac{7}{1}$$

$$200(r-30) + 60r = 7r(r-30)$$

$$200r - 6000 + 60r = 7r^2 + 210r = 0$$

$$7r^2 - 470r + 6000 = 0$$

$$7r - 120 = 0$$

$$r - 50 = 0$$

$$7r = 120$$

$$r = 50 \text{ Rate outside}$$

$$r = 17\frac{1}{7}$$

$$r - 30 = -12\frac{6}{7}$$

Reject

On a trip of 480 miles the rate of an airplane was increased 20 mph by a "tail wind". Returning it was decreased by the same amount. If the total time for the trip was 7 hours, at what rate in calm air was the airplane flying under its own power?

	GOING	RETURNING
DISTANCE	480	480
RATE	$r + 20$	$r - 20$
TIME	$\frac{480}{r+20}$	$\frac{480}{r-20}$

Let $r =$ rate of plane in calm air

$$\frac{480}{r+20} + \frac{480}{r-20} = 7$$

$$M (r+20)(r-20) \quad 480(r-20) + 480(r+20) = 7(r+20)(r-20)$$

$$480r - 9600 + 480r + 9600 = 7r^2 + 2800 = 0$$

$$7r^2 + 960r - 2800 = 0$$

$$7r + 20 = 0 \quad r - 140 = 0$$

$$7r = -20 \quad r = 140$$

$$r = -2\frac{2}{7} \quad \text{Rate in calm air}$$

REJECT

The rate of one train is 15 mph more than that of another train. If the faster train takes one hour less than the other for a trip of 180 miles, at what rate does each of the train travels?

	SLOWER	FASTER
DISTANCE	180	180
RATE	r	$r + 15$
TIME	$\frac{180}{r}$	$\frac{180}{r + 15}$

Let $r =$ rate of the slower in mph

$$r + 15 = \text{ " " " faster " " }$$

$$\frac{180}{r + 15} + 1 = \frac{180}{r}$$

$$M_A(r + 15) \quad 180r + r(r + 15) = 180(r + 15)$$

$$180r + r^2 + 15r = 180r + 2700$$

$$-r + 15r - 2700 = 0$$

$$(r + 60)(r - 45) = 0$$

$$r + 60 = 0 \quad r - 45 = 0$$

$$r = -60 \quad r = 45 \text{ rate of slower}$$

$$\text{REJECT } r + 15 = 60 \text{ " " faster}$$

At what speed in mph must an aeroplane be flown to make a round trip in 7 hrs. between two points that are 600 miles apart, if it must travel with a 25 mile wind when going and against it when returning?

	GOING	RETURNING
DISTANCE	600	600
RATE	$x + 25$	$x - 25$
TIME	$\frac{600}{x+25}$	$\frac{600}{x-25}$

$$\frac{600}{x+25} + \frac{600}{x-25} = \frac{7}{1}$$

$$M \quad (x+25)(x-25) \quad 600(x-25) + 600(x+25) = 7(x+25)(x-25)$$

$$600x - 15,000 + 600x + 15,000 = 7x^2 - 4375$$

$$\overset{(+)}{-} 7x^2 \overset{(-)}{+} 1200x \overset{(-)}{+} 4375 = 0 \quad x - 175 = 0$$

$$(7x+25)(x-175) = 0 \quad x = 175$$

$$7x + 25 = 0$$

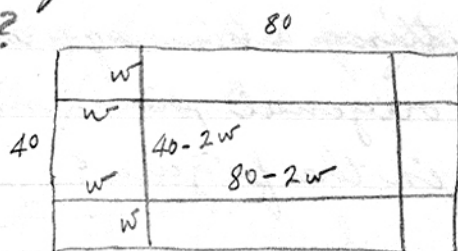
$$7x = -25$$

$$x = -3\frac{4}{7}$$

REJECT

Speed in mph

12. How wide a border must a farmer cut around the outside of a field of grain that is 40 rods wide and 80 rods long in order that he shall have cut $\frac{1}{5}$ of the field?



Let w = the width

$$(40 - 2w)(80 - 2w) = \frac{1}{5}(40)(80)$$

$$3200 - 240w + 4w^2 - 640 = 0$$

$$4w^2 - 240w + 2560 = 0$$

$$w^2 - 60w + 640 = 0$$

$$w = \frac{-(-60) \pm \sqrt{3600 - 4(1)(640)}}{2(1)}$$

$2(1)$

$$w = 30 \pm 2\sqrt{65}$$

$$w = 30 - 2\sqrt{65}$$

$$w = 30 + 2\sqrt{65} = 30 + 2(8.062)$$

$$w = 30 - 2(8.062)$$

$$w = 30 + 16.124$$

$$w = 30 - 16.124$$

$$w = 46.124$$

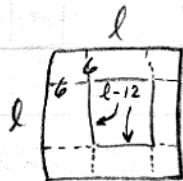
$$w = 13.876$$

REJECT

$$w = 13.9 \text{ Width}$$

69.

How long must the sides of a square piece of cardboard be in order that the box made from it shall contain 864 in^3 , if the box is made by cutting a 6 in. square from each corner of the original piece and bending it up as given in the figure?



Let l = length of side in inches

$$6(l-12)^2 = 864$$

$$\therefore (l-12)^2 = 144$$

$$l^2 - 24l + 144 - 144 = 0$$

$$l^2 - 24l = 0$$

$$l(l-24) = 0$$

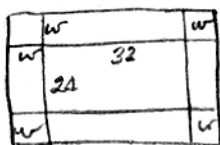
$$l = 0$$

$$l - 24 = 0$$

reject

$l = 24$ Length of side in inches

A picture is 24 in. wide and 32 in. long. How wide, correct to the nearest inch, must a frame around it be so that the area of the picture itself will be $\frac{2}{3}$ of the total surface covered the picture and its frame?



Let w = width of frame in inches

$$24(32) = \frac{2}{3}(24 + 2w)(32 + 2w)$$

$$\text{Ans } 72(32) = 2(768 + 112w + 4w^2)$$

$$2304 = 1536 + 224w + 8w^2$$

$$8w^2 + 224w - 768 = 0$$

$$\text{Div } w^2 + 28w - 96 = 0$$

$$w = \frac{-28 \pm \sqrt{784 - 4(1)(-96)}}{2(1)} = \frac{-28 \pm \sqrt{784 + 384}}{2}$$

$$w = \frac{28 \pm \sqrt{1168}}{2} = \frac{-28 \pm 4\sqrt{73}}{2}$$

$$w = -14 \pm 2\sqrt{73}$$

$$w = -14 + 2\sqrt{73}$$

$$w = 14 + 2(8.544)$$

$$w = 14 + 17.088$$

$$w = 3.088$$

$$w = 3 \text{ Width in inches}$$

$$w = -14 - 2\sqrt{73}$$

$$w = -14 - 2(8.544)$$

$$w = -14 - 17.088$$

$$w = -31.088$$

REJECT

The numerator of a certain fraction is 3 less than its denominator. The sum of this fraction and the one formed by adding 1 to both its terms is $\frac{9}{10}$. What is the fraction?

Let d = denominator

$d - 3$ = numerator

$\frac{d-3}{d}$ } the fraction

$$\frac{d-3}{d} + \frac{d-2}{d+1} = \frac{9}{10}$$

M. $10d(d+1)$

$$10(d+1)(d-3) + 10d(d-2) = 9d(d+1)$$

$$10(d^2 - 2d - 3) + 10d^2 - 20d = 9d^2 + 9d$$

$$10d^2 - 20d - 30 + 10d^2 - 20d - 9d^2 - 9d = 0$$

$$11d^2 - 49d - 30 = 0$$

$$(d-5)(11d+6) = 0$$

$$d-5=0 \quad 11d+6=0$$

$$d=5 \quad 11d=-6$$

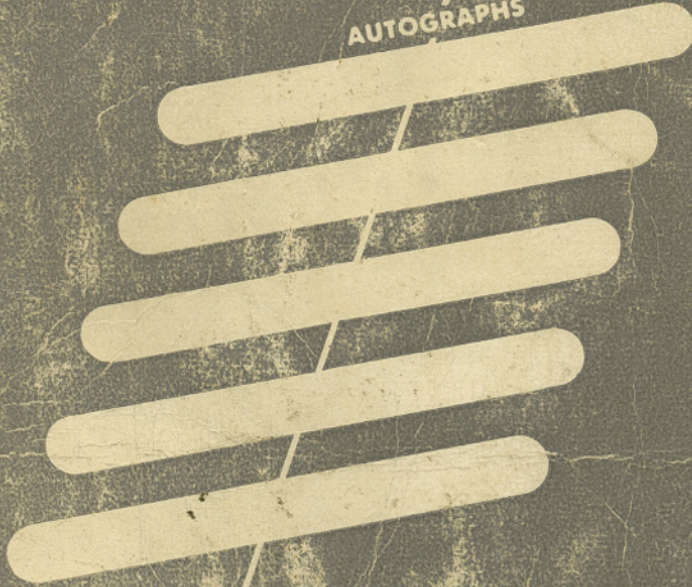
$$d-3=2 \quad d \text{ etc}$$

$$\frac{d-3}{d} = \frac{2}{5}$$

REJECT (does not fit)

Fraction

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